Calculus BC

<u>Section 1.3 - Evaluating Limits Analytically</u>

<u>Obj:</u> - Evaluate a limit using properties of limits
- Develop and use a strategy for finding limits
- Evaluate a limit by dividing out and rationalizing
- Evaluating a limit using the Squeeze Theorem

Given $\lim\limits_{x\to c} f(x) = L$ and $\lim\limits_{x\to c} g(x) = K$

- $\lim\limits_{x\to c} b\cdot f(x) = bL$ — <u>Scalar Multiple</u>

  where b is a constant

- $\lim\limits_{x\to c} [f(x) \pm g(x)] = L \pm K$ — <u>Sum/Difference</u>
- $\lim\limits_{x\to c} [f(x)\cdot g(x)] = LK$ — <u>Product</u>
- $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{K}$ — <u>Quotient</u>
- $\lim\limits_{x\to c} [f(x)]^n = L^n$ — <u>Power</u>
- $\lim\limits_{x\to c} k = k$ — <u>Constant</u>

Given polynomial $P(x)$,

- $\lim_{x \to c} P(x) =$

-<u>Strategies</u> for finding $\lim_{x \to c} f(x)$:

a) direct substitution: $f(c)$

b) if $f(c)$ is undefined, simplify $f(x)$ by factoring, reducing, rationalizing, using conjugates, ….

<u>Definition:</u>

A limit that results in $\frac{0}{0}$ or $\frac{\infty}{\infty}$ is <u>indeterminate.</u>

Try strategy b above.

1. Find $\lim_{x \to 1} \frac{x^2 - 4}{x - 2}$

-direct substitution

2. Find $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

-direct substitution:

-undefined at x = 2,
try simplifying f(x)

3. Find $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x}$

-direct substitution yields ____

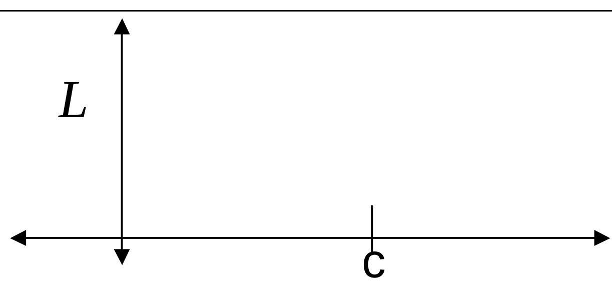
-mult. by the ___________

- try finding the limit again

The Squeeze Theorem:

Given $g(x) \le f(x) \le h(x)$ in some interval about c for all $x \ne c$
and $\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$

then $\lim_{x \to c} f(x) = L$



From the Squeeze Theorem, we have:

$\lim_{x \to 0} \frac{\sin x}{x} = 1$	$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

4. $\lim_{x \to 0} \frac{x\cos x}{\sin x} =$

- separate $\frac{x}{\sin x}$

- replace with $\frac{\sin x}{x}$

- take limit

5. $\lim_{x \to 0} \frac{\tan x}{x} =$

- in terms of sin x

- separate

6. $\lim_{x \to 0} \frac{\sin 5x}{2x}$

7. $\lim_{x \to \infty} x^2 \sin\frac{1}{x} =$

-note limit of x

- let $\theta =$

-change everything to θ