Calculus BC

<u>Section 1.4 - Continuity and One-Sided Limits</u>

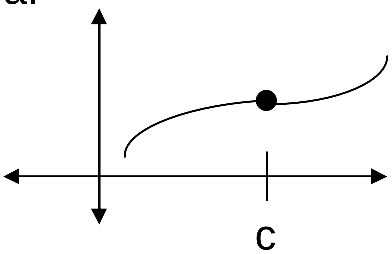
<u>Obj:</u> - Determine continuity at a point and continuity on an open interval
- Determine one-sided limit and continuity on a closed interval
- Use properties of continuity
- Understand and use the Intermediate Value Theorem

Definition of Continuity
A function f is **<u>continuous</u>** at point c if the following three conditions are met: 1. $f(c)$ is defined 2. $\lim_{x\to c} f(x)$ exists, that is, $\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x)$ 3. $f(c) = \lim_{x\to c} f(x)$

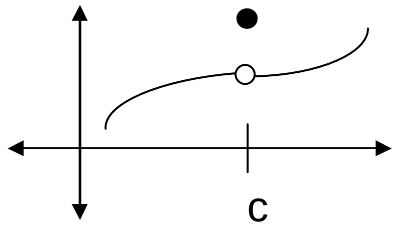
Note: on a closed interval $[a,b]$, a function can have a one-sided limit at the endpoints to satisfy condition 2.

1. State whether the following are continuous at x = c.
   If not, state the condition of continuity in which it fails.

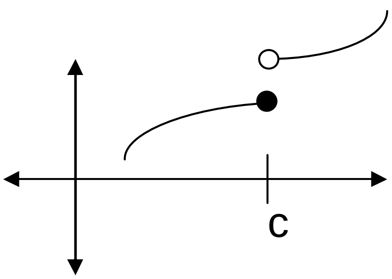
a.



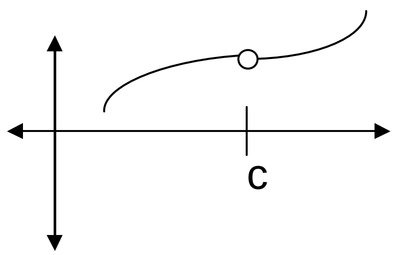
b.



c.



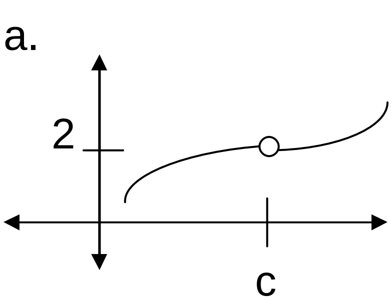
d.



Definition:

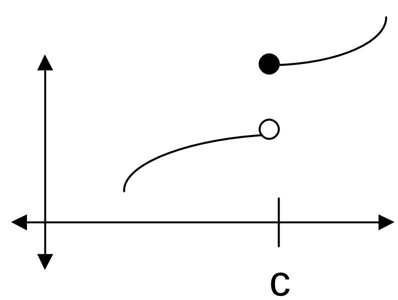
If $f(x)$ is not continuous at c, it either has a **<u>removable discontinuity</u>** or a **<u>nonremovable discontinuity</u>**

a.



removable
can redefine $f(c)$ to equal 2
open circle will be closed

b.



nonremovable

2. Define $f(7)$ in a way that extends $f(x)=\frac{x^2-49}{x-7}$ to be continuous at $x=7$.

- function is discontinuous at _____

-find equivalent function

-evaluate $f(7)$

new piecewise function: $$f(x)=\begin{cases}\frac{x^2-49}{x-7} & x\neq 7 \\ & x=7\end{cases}$$

3. State the interval of continuity of each function.

a) $f(x)=\frac{1}{x-2}$

-parent graph _____

b) $g(x)=\frac{x^2-4}{x-2}$

-find domain

-find equivalent function

c) $h(x)=\begin{cases} x+1 & x<0 \\ x^2+1 & x>0 \end{cases}$

-graph the piecewise function

Continuity of Polynomial Functions and Rational Functions

- all polynomial functions are continuous at any point $x=c$.
- If $f(x)$ and $g(x)$ are continuous then the following are continuous:

  $f \pm g$, fg, kg, $f(g(x))$, $g(f(x))$ where k is a constant

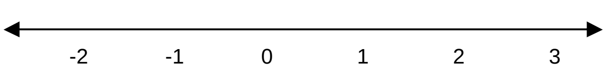
  $\frac{f}{g}$ is continuous at every point where $g(x) \neq 0$

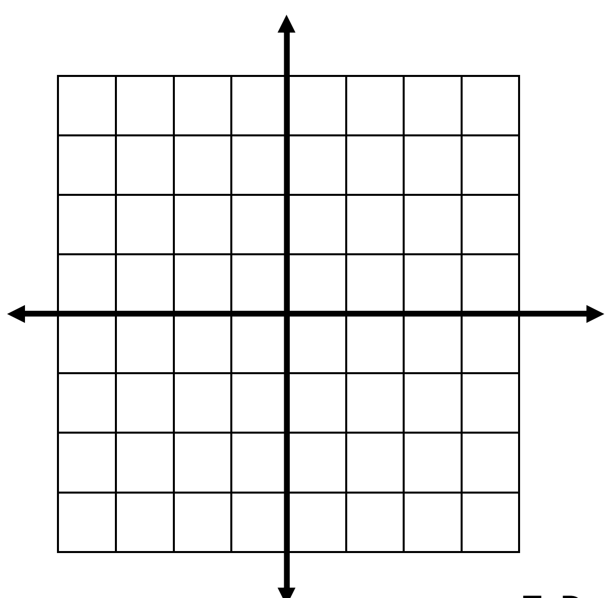
Greatest Integer Function: $f(x)=[x]$

Given x, $f(x)$ is the greatest integer that is less than or equal to x

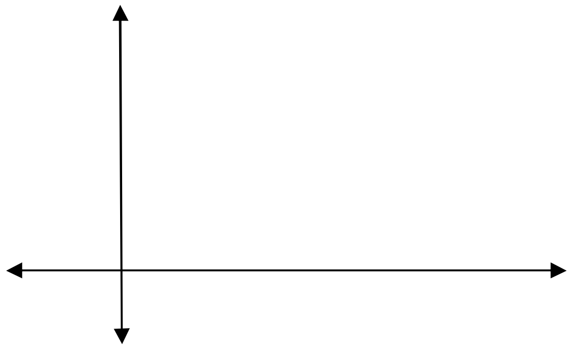
-make a table, a number line will also be helpful

x	f(x)





Theorem: **Intermediate Value Theorem**
If f is continuous on the closed interval $[a,b]$, $f(a) \neq f(b)$, and $f(a) < k < f(b)$, then there is **at least** one value c in $[a,b]$ such that $f(c) = k$



Consider a car accelerating from 0 to 60 mph in 5 seconds,
$t = 0$ represents ____, $t = 5$ represents ____
0 mph represents ___, 60 mph represents ____

Since the velocity is continuous, there is a point in <u>time</u> when the odometer reads 50 mph.

50 mph represents ____, the point in time when this occurs is the value *c.*

-The Intermediate Value Theorem often can be <u>used to locate the zeros</u> of a function that it continuous on a closed interval:

given the continuous funtion f in interval [a,b].

If $f(a)$ and $f(b)$ differ in sign (one is pos, one is neg), then somewhere in the interval [a,b], there is at least one value c such that $f(c) = 0$