

Calculus BC

Section 3.2 - Rolle's Theorem and the Mean Value Theorem

Obj: - To understand and use Rolle's Theorem.
- To understand and use the Mean Value Theorem.

Rolle's Theorem

Given $f(x)$ continuous in the closed interval $[a, b]$ and differentiable in the open interval (a, b) .

If $f(a) = f(b) = 0$, then there exists at least one c in (a, b) such that $f'(c) = 0$

Graphical representation of theorem.

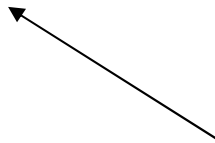
The Mean Value Theorem

If $f(x)$ is continuous in the closed interval $[a, b]$ and differentiable in the open interval (a, b) , then \exists at least one c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



slope of tangent
at $x=c$



slope of segment
containing endpoints

The 'mean' refers to the mean (average) rate of change of the interval.

-Graphical representation of theorem.

Geometrically, this theorem guarantees the existence of a tangent line that is parallel to the secant containing the endpoints.

1. Using the Mean Value Theorem, find c given

$$f(x) = x^2 + 4x - 5 \text{ and } x \in [1,3]$$

Make sure the hypotheses of the theorem are satisfied so that MVT can be applied.

-Is $f(x)$ continuous on $[1, 3]$? that is, is $f(x)$ defined for every point in the closed interval?

- Is $f(x)$ differentiable on $(1, 3)$? that is, is $f'(x)$ defined for every point in the open interval?

-Can the MVT be applied?

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

-apply MVT

2. Are the hypotheses of the Mean Value Theorem satisfied for $f(x) = \sqrt{x}$ where $x \in [0,2]$?

a) Is $f(x)$ continuous in the closed interval $[0,2]$?
(Does $f(x)$ exist at every point in the closed interval?)

b) Is $f(x)$ differentiable in the open interval $(0,2)$?
(Does $f'(x)$ exist for every point in the open interval?)

-Recall the **Intermediate Value Theorem**:

If $f(x)$ is continuous on $[a,b]$ then it takes on every y value between $f(a)$ and $f(b)$.

From this theorem we get:

If $f(a)$ and $f(b)$ have opposite signs, then there is at least one c in (a,b) such that $f(c) = 0$

(since $y = 0$ is an intermediate value of $f(a)$ and $f(b)$)

Combine this theorem with the following to answer example 3:

- graph goes up if $f'(x) > 0$

- graph goes down if $f'(x) < 0$

3. Show that $f(x) = x^3 + 3x + 1 = 0$ has exactly one solution in $[-1, 1]$.

Intermediate Value Theorem:

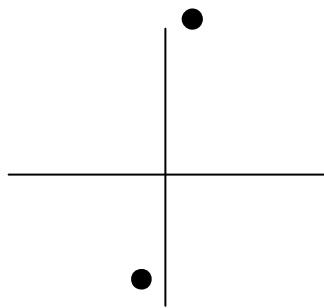
-find $f(a)$

-find $f(b)$

Since the y values have _____ signs, there is **at least** one zero in this interval (there is a point c where $f(c) = 0$).

Derivative increasing/decreasing:

$f'(x) = 3x^2 + 3$ is always positive in the interval $[-1, 1]$ so graph is always increasing.



there is exactly ____ solution in $[-1, 1]$

If $f'(x) = 2x^2 - 4$ for the interval $[-1, 1]$

-set $f'(x) = 0$, solve

-number line, signs