Calculus BC

Section 3.4 - Concavity and the Second Derivative Test

Obj: - To use the second derivative to determine whether a function is concave upward or downward.

Concavity:

- $f''(x) > 0 \Rightarrow$ graph of f is concave upward 
- $f''(x) < 0 \Rightarrow$ graph of f is concave downward 

Point of Inflection

The point $(c, f(c))$ is a point of inflection of f if there is a change in concavity from positive to negative (or negative to positive) at that point.

-this textbook also requires the existence of the tangent at the point $(c, f(c))$

for example $f(x) = \begin{cases} x^3 & x < 0 \\ x^2 + 2x & x \geq 0 \end{cases}$ is not considered to have a point of inflection at $x = 0$.

change of concavity:

existence of the tangent line:

1. Find the point of inflection (IB – inflexion) and discuss the concavity.
   $f(x) =$

-find $f'(x)$

-solve $f''(x) = 0$

-determine test intervals

-apply test values to find sign of f''

point of inflection:

concavity:

<u>Theorem: The Second Derivative Test</u>

Given function f with $f'(c) = 0$ and f'' exists in the interval containing c

- $f''(x) > 0 \Rightarrow f$ has a relative ________ at $(c, f(c))$
- $f''(x) < 0 \Rightarrow f$ has a relative ________ at $(c, f(c))$

2. Find all relative extrema of $f(x) = -x^4 + 4x^3 + 8x^2$ using the Second Derivative Test.

-find $f'(x)$

-find critical values by solving ______=0

-find $f''(x)$

-evaluate f'' at each critical value

If $f'(c)$ is undefined, then $f''(c)$ ____________