

Section 4.4 - The Fundamental Theorem of Calculus

- Obj:
- To evaluate a definite integral using the Fundamental Theorem of Calculus.
 - To understand and use the Mean Value Theorem for Integrals.
 - To find the average value of a function over a closed interval.
 - To understand and use the Second Fundamental Theorem of Calculus.

The Fundamental Theorem of Calculus

If f is continuous on $[a, b]$ and F is the antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Definite Integral

1. $\int_1^3 x^2 dx =$

- find the antiderivative of x^2 , call this F
- find $F(3) - F(1)$

$$2. \text{ a) } \int_0^2 x^2 dx + \int_2^4 x^2 dx =$$

$$\text{b) } \int_0^4 x^2 dx =$$

-Conclusion from parts (a) and (b):

$$3. \int_0^4 (x^2 - 4x + 5) dx =$$

5. $\int_0^{\pi} \sin x dx$

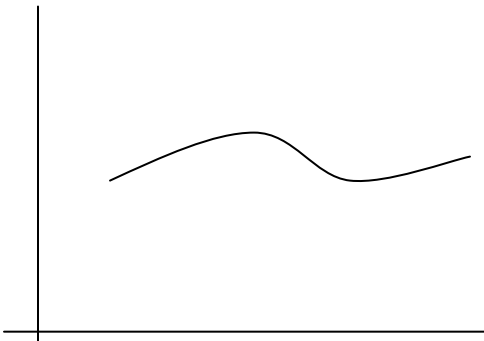
6. $\int_1^9 \frac{x-1}{\sqrt{x}} dx$

HW p. 293 # 9, 15, 17, 21, 27, 29

Section 4.3 - Riemann Sums and Definite Integrals

- Obj:
- To understand the definition of a Riemann sum
 - To evaluate a definite integral using limits
 - To evaluate a definite integral using properties of definite integrals.

Finding area under a curve (positive $f(x)$)



We can approximate the area if we divide into n equal intervals with the width of Δx

- As the number of intervals n (rectangles) increases, what happens to the approximation?
- The intervals do not need to have the same width Δx . They could have different widths, Δx_k .

So we could express the area as:

$$A \approx \Delta x_1 \cdot f(c_1) + \Delta x_2 \cdot f(c_2) + \Delta x_3 \cdot f(c_3) + \dots + \Delta x_n \cdot f(c_n)$$

or, in summation notation:

$$A \approx \sum$$

Riemann sum for f on $[a, b]$

- As $n \rightarrow \infty$, we have an infinite number of intervals, the area is then written as:

$$A = \sum =$$

Definite Integral as the Limit of Riemann Sums or
Riemann Integral

Parts of an Integral: $\int_a^b f(x)dx$

- \int integral sign – Leibniz choose sign to resemble the S in German word for summation
- read as: the integral of f from a to b
- a and b are the lower and upper limits, respectively.
- x is the variable of integration
- $f(x)$ is the integrand

We could also have:

$$\int_a^b f(t)dt$$

$$\int_a^b f(s)ds$$

$$\int_a^b ydy$$

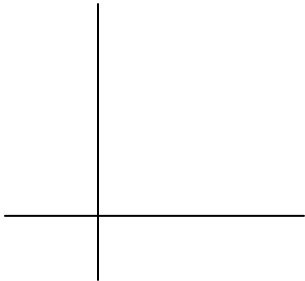
Theorem:

If f is continuous on $[a, b]$, then f is integrable on $[a, b]$,

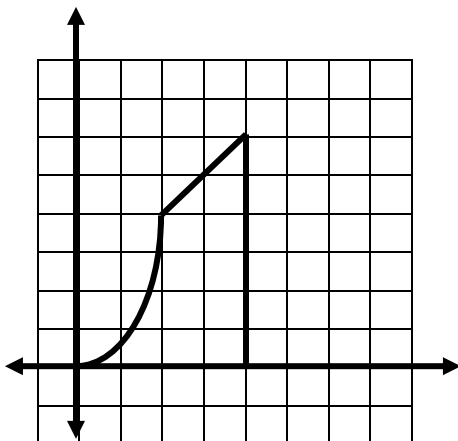
that is, $\int_a^b f(x)dx$ exists.

1. Write the definite integral to represent the area bounded by $y = x + 2$, $x = 0$, $x = 2$ and the x-axis. Then evaluate the integral.

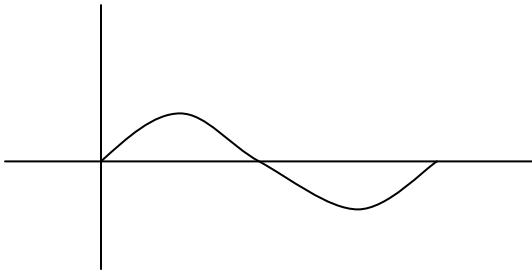
2. Write the definite integral to represent the area bounded by $y = x^2 + 1$, $x = 0$, $x = 2$ and the x-axis. Then evaluate the integral.



3. Express the area of the shaded region using definite integrals

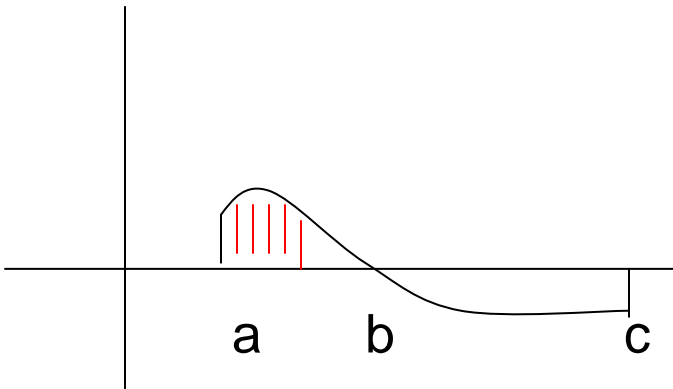


5. Find the shaded area



$$y = \sin x$$

6. Write the integral(s) to find the shaded region



i)

ii)

iii)

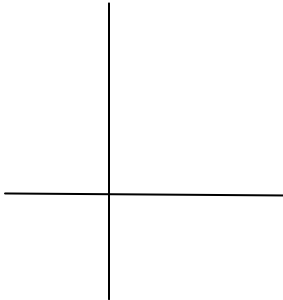
p. 279 # 13-21 odd, 41, 43, 44, 52-54, 77

p. 293 # 35, 36, 39, 41

Section 4.4 – The Fundamental Theorem of Calculus (continued)

Definite integral involving absolute value.

1. Evaluate $\int_{-1}^5 |x - 2| dx$



-examine graph

-rewrite as two separate equations without absolute value

-Write as two separate integrals with new corresponding limits

2. Evaluate $\int_0^{3\pi/2} |\cos x| dx$

3. Let $F = \int_a^x (t^2 + 2t + 3)dt$. Find $\frac{dF}{dx}$.

a is a constant

The Second Fundamental Theorem of Calculus

If f is continuous on an open interval containing a , then

$$\frac{d}{dx} \left[\int_a^x f(t)dt \right] =$$

taking the derivative of an integral

4. Find $\frac{d}{dx} \int_3^x (t^4 + 2t)dt$

5. Find $\frac{d}{dx} \int_7^x (\sin t + \cos t) dt$

6. Find $\frac{dF}{dx}$ if $F = \int_0^{x^2} t dt$

7. Find $\frac{d}{dx} \int_7^{x^2} (\sin t + \cos t) dt$

8. Find $\frac{d}{dx} \int_{x^2}^7 t dt$

9. Find $\frac{d}{dx} \int_{x^2}^{x^3} t dt$

Extension to Second Fundamental Theorem:

$$\frac{dF}{dx} = \frac{d}{dx} \int_{g(x)}^{h(x)} f(t) dt =$$

Average Value of an integrable function f on $[a, b]$ is

$$\frac{1}{b-a} \int_a^b f(x) dx$$

(this gives the average y value in the interval)

10. Find the average value of the function $f(x) = x^2 + 1$ on the interval $[1, 4]$.

Mean Value Theorem for Definite Integral:

If f is continuous on $[a, b]$, then at some point c in $[a, b]$,

$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx \quad (\text{average value})$$

The Net Change Theorem

The definite integral of the rate of change of a quantity $F'(x)$ gives the total change (net change) in that quantity on the interval $[a, b]$.

$$\int_a^b F'(x) dx = F(b) - F(a)$$

Net change of F

<u>Displacement</u> on $[a, b] = \int_a^b$
Total <u>distance traveled</u> on $[a, b] = \int_a^b$

11. A particle is moving along a line with velocity

$$v(t) = t^2 - 3t.$$

a) Find the displacement of the particle on the time interval $1 \leq t \leq 6$.

b) Find the total distance traveled on the time interval $1 \leq t \leq 6$.