

# Calculus BC

## Section 5.5 - Bases Other than e and Applications

- Obj: - To find the derivative and integral of  $y = a^x$ .  
- To use exponential functions to model compound interest and exponential growth.

### Derivative of $a^x$ :

-We know that the derivative of  $x^a$  is \_\_\_\_\_  
(  $x$  is the variable,  $a$  is a rational number)

-What about the derivative of  $a^x$  ?

variable

constant

start with  $y = a^x$

-take the  $\ln$  of both sides

-bring  $x$  down

-differentiate

-solve for  $\frac{dy}{dx}$

$$\frac{d}{dx} a^x =$$

$$\frac{d}{dx} a^u =$$

1.  $y = 5^{x^2}$  find  $\frac{dy}{dx}$

2.  $y = 3^{\cos x}$  find  $\frac{dy}{dx}$

3.  $y = x^{2x}$  find  $\frac{dy}{dx}$

(variable in both the base and exponent)

-need to bring the x from  
the exponent down, take ln

**Integral of  $a^x$ :**

$$a^x = e^{x \ln a}$$

-integrate both sides  
let  $u =$

$$\int a^x dx =$$

4.  $\int 3^{4x} dx$

5.  $\int 4^{\cos x} \sin x dx$

## Base "a" Logarithm – Derivative and Integral

$\log_a x$  is the inverse of \_\_\_\_\_

$$\log_a a^x =$$

$$a^{\log_a x} =$$

-To find the derivative and integral, we change  $\log_a x$  to  $\ln$

$$a^{\log_a x} =$$

-take the  $\ln$  of both sides

-remove exponent

-solve for  $\log_a x$

$\log_a x =$

This is the \_\_\_\_\_ formula

### The Derivative of $\log_a x$ :

$\frac{d}{dx}(\log_a x) =$

$\frac{d}{dx}(\log_a u) =$

6. Find  $\frac{d}{dx} \left( \log_3(3x^2 + 1) \right)$

-change of base to ln

-differentiate

7.  $\int \frac{\log_2 x}{x} dx$

-change of base to ln

-integrate

## **Compound Interest Formula**

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

$A$  - balance after  $t$  years

$P$  - principle

$r$  - annual rate

$n$  - number of compounding per year

## **Theorem – Limit involving $e$**

$$\lim_{x \rightarrow \infty} \left( 1 + \frac{1}{x} \right)^x = e$$

8. \$5000 is invested at an annual rate of 8%.
- a) Find the balance at the end of 10 years if the account compounds monthly.
- b) Find the balance at the end of 10 years if the account compounds continuously.

## Logistic Growth Function

9. The weight of a bacterial culture grows according to the logistic growth function  $w(t) = \frac{1.25}{1 + 0.25e^{-0.4t}}$ .

a) find  $w(0)$

b) find  $w(1)$

c) find  $w(10)$

d) find  $\lim_{t \rightarrow \infty} w(t)$

e) what the the horizontal asymptote of  $w(t)$  ?