Calculus BC

Section 6.3 - Separation of Variables and the Logistic Equation

Obj: - To recognize and solve differential equations by separation of variables.
- To recognize and solve homogeneneous differential equations.

Definition:

A first order differential equation is **separable** if it can be put in the form

$M(x)dx + N(y)dy = 0$ or $M(x) + N(y)\dfrac{dy}{dx} = 0$

1. Find the general solution of $-xy + (x^2 + 4)\dfrac{dy}{dx} = 0$

-separate the variables

-integrate

Definition:

A **homogeneous differential equation** is of the form

$$M(x,y)dx + N(x,y)dy = 0$$

where M and N are homogeneous functions of the same degree.

Solving a homogeneous differential equation-

Substitute $y = vx$ where v is a function of x

Substitute $dy =$

This will yield a separable differential equation in v and x

2. Solve $(x^2 - y^2)dx + 3xydy = 0$

-cannot separate x and y

-let $y = vx$

$dy =$

-substitute, eq'n will be in terms of v and x

-separate variables

-integrate

-rewrite in terms of x and y

Logistic Growth

Consider the exponential growth $y = Ce^{kt}$, what is the limiting value?

Realistically, there must be an upper limit L, the carrying capacity.

A model to describe the type of growth with a limiting capacity is the **logistic differential equation**

$$\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$$

3. Solve the logistic differential equation $\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$.

4. A game refuge initially contains 40 elks. After 5 years, there are 104 elks. The environment can support up to 4000 elks.

a) Write the logistic differential equation then solve for the particular solution.

$$\frac{dp}{dt} =$$

$p =$ $L =$ $p(0) =$

We can apply the general solution from problem 3

$p =$

b) Calculator - Graph the slope field $\frac{dp}{dt} =$

and the particular solution that contains the initial condition $p(0) =$

Look at the limiting value. $p =$

c) Use the particular solution from part (a), estimate the population when $t = 15$.

d) Find the limit when $t \to \infty$