Calculus BC

Section 9.2 - Series and Convergence

Obj: - To identify a geometric series and find the sum of convergent ones.
- To use the nth-Term Test for Divergence

Geometric Series:

$$a + ar + ar^2 + ar^3 + ar^4 + ... + ar^{n-1} + ...$$

- Finding the sum of n geometric terms:

$S_n =$

$rS_n =$

$(1 - r)S_n =$

☺

Partial Sum of a Geometric Series	
$S_n =$	or $S_n =$

Finding the sum of a geometric series: (sum of all the terms)

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{a(1-r^n)}{1-r}$$

- If $|r|<1$ then $\lim_{n\to\infty} S_n =$

- If $|r|\geq 1$ (such as $A=2+4+8+16+....$ where $r=2$)

  then $\lim_{n\to\infty} S_n$ =

☺

Sum of a Geometric Series

$S =$

1. Find the sum of the series $\sum_{1}^{\infty}\left(\frac{1}{3}\right)^{n+1}$

   - this is a ____________ series

   - first term $a =$, ratio $r =$
   - since $|r|<1$ then series ___verges

   $S =$

2. Find the sum of the series $\sum_{1}^{\infty}\left(\frac{3}{2}\right)^{n+1}$

- this is a ____________ series
- first term $a =$, ratio $r =$
- since $|r|$ 1, series ___verges

$S =$

3. Does the series $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ converge?

If yes, find the sum.

$$\frac{1}{n(n+1)} =$$

- apply partial fractions

$$\sum_{n=1}^{\infty}\frac{1}{n(n+1)} = \sum_{n=1}^{\infty}\left(\frac{}{} - \frac{}{}\right)$$

=

Definition of Convergent and Divergent Series

An infinite series <u>converges</u> if its partial sums S_n converges to a value S

An infinite series <u>diverges</u> if its partial sums S_n diverges

☺☺ **The n^{th}-Term Test for Divergence**

If $\lim_{n\to\infty} a_n \neq 0$ or if $\lim_{n\to\infty} a_n$ fails to exist

then $\sum_{n=1}^{\infty} a_n$ **<u>diverges</u>**

If $\sum_{n=1}^{\infty} a_n$ <u>converges,</u> then $\lim_{n\to\infty} a_n = 0$

<u>Note:</u> avoid the following common mistake:

$\sum_{n=1}^{\infty} a_n$ converges $\Rightarrow$ $\lim_{n\to\infty} a_n = 0$

$$1+\underbrace{\frac{1}{2}+\frac{1}{2}}_{\text{2 terms}}+\underbrace{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}_{\text{4 terms}}+.....+\underbrace{\frac{1}{2^n}+\frac{1}{2^n}+...+\frac{1}{2^n}}_{\text{n terms}}+...$$

= 1 + 1 + 1 + ….. + 1 + ….

=

this series **diverges** even though $a_n \to 0$

4. State whether the following series diverge or converge

a) $\sum_{n=1}^{\infty} \frac{n+1}{n}$

$a_n \to$ _____ , that is, $\lim_{n\to\infty} a_n \neq$ ___

so series _____verges by ____________________________

(reason)

b) $\sum_{n=1}^{\infty} (-1)^{n+1}$

$a_n \to$ _____ , that is, $\lim_{x\to\infty} a_n$ ___

so series _____verges

c) $\sum_{n=1}^{\infty} \frac{n}{2n+5}$

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{n}{2n+5} =$$

so series _____verges by __________________________

d) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 5}{4^n}$

-list a few terms of the series