**Calculus BC**

## Section 9.3 - The Integral Test and p-Series

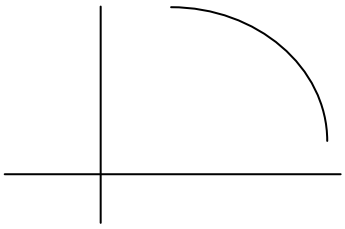
Obj: - To use the Integral Test to determine convergence or divergence of a series.
- To use properties of p-series and harmonic series.

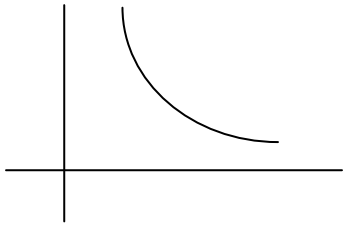
### ☺ The Integral Test

If $f(x)$ is a continuous, positive, decreasing function for $x \geq 1$, and $a_n = f(n)$, then

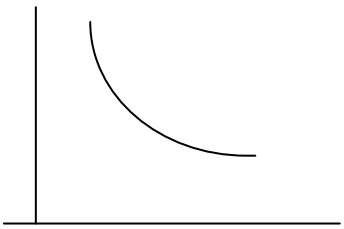
$$\sum a_n \quad \text{and} \quad \int_1^{\infty} f(x)dx$$

will either both diverge or both converge





(Think first quadrant and decreasing)



$$\sum a_n = \sum_1^n f(i)$$

1. Does the series $\sum_{1}^{\infty} \frac{1}{n^2}$ converge or diverge?

Examine $f(x) = \frac{1}{x^2}$.

Is it continuous, positive, and decreasing for $x \geq 1$?

Integrate to see if $\int_{1}^{\infty} f(x)dx = \int_{1}^{\infty} \frac{1}{x^2} dx$ converges.

Since $\int_{1}^{\infty} f(x)dx = \int_{1}^{\infty} \frac{1}{x^2} dx$ ___verges, then

by the _____ Test, the series $\sum_{1}^{\infty} \frac{1}{n^2}$ ___verges

The **Harmonic Series** $\sum_{1}^{\infty}\frac{1}{n}$

$$\sum_{1}^{\infty}\frac{1}{n}=1+\frac{1}{2}+\left(\frac{1}{3}+\frac{1}{4}\right)+\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\right)+\left(\frac{1}{9}+\frac{1}{10}+\ldots+\frac{1}{16}\right)+\ldots$$

in grouping the terms, each group gives a sum that is $>\frac{1}{2}$

$$\sum_{1}^{\infty}\frac{1}{n} > \frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\ldots+\frac{1}{2}+\ldots.$$

Therefore,

☺ the **Harmonic Series** $\sum_{1}^{\infty}\frac{1}{n}$ ____verges

☺ **The p-series**

$$\sum_{n=1}^{\infty}\frac{1}{n^{p}} = \frac{1}{1^{p}}+\frac{1}{2^{p}}+\frac{1}{3^{p}}+\frac{1}{4^{p}}+\frac{1}{5^{p}}+\ldots\ldots$$

- converges if $p>1$
- diverges if $p\le 1$

Can be proven using the Integral Test.