Calculus BC

Section 9.4 - Comparison of Series

Obj: - To use the Direct Comparison Test to determine the convergence or divergence of a series.
- To use the Limit Comparison test to determine the convergence or divergence of a series.

- We can show that a series converges/diverges by comparing it to another series that is known to converge/diverge.

1. Determine the convergence of $A = \sum_{0}^{\infty} \frac{1}{3+2^n}$.

Solution:

This is similar to the series $B = \sum_{0}^{\infty} \frac{1}{2^n}$

Compare the series, term by term:

$$A = \sum_{0}^{\infty} \frac{1}{3+2^n} =$$

$$B = \sum_{0}^{\infty} \frac{1}{2^n} =$$

Since we know that series B ___verges by ______________ and that a_n ____ b_n for every n, we can conclude by the Direct Comparison Test that series A ___verges.

2. Determine the convergence of

$$A = \sum_{0}^{\infty} \frac{1}{n!} = 1 + 1 + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \ldots$$

Solution:

-Consider the series $B = \sum_{0}^{\infty} \frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \ldots$

-We know that series B ___verges because it is a __________ series with $r =$____

-Compare each term of series A to each term of series B:

a_n ____ b_n for every n (with the exception of the first term)

So we can modify series B to be $B = 1 + \sum_{0}^{\infty} \frac{1}{2^n}$

-Since series B ___verges, series A also ___verges.

Note: To find a series B to compare, take the limit of a_n as $n \to \infty$ but keep the n so it can be used in series B.

3. Determine the convergence of $A=\sum_{0}^{\infty}\frac{1}{-5+\sqrt[3]{n}}$.

-Compare to series B=

☺ **<u>The Limit Comparison Test</u>**

If $a_n > 0,\ b_n > 0,$ and

$$\lim_{n\to\infty}\frac{a_n}{b_n}=L$$

where L is finite and positive, then

the two series either <u>both diverge</u> or <u>both converge</u>

as $n\to\infty,\ a_n = Lb_n$ so $\sum_{n=1}^{\infty}a_n = L\sum_{n=1}^{\infty}b_n$

so if one diverges, the other, its multiple, will also diverge.

4. Does $A = \sum_{2}^{\infty} \frac{2n}{n^2 - n + 1}$ converge or diverge?

- Find a series *B* to compare:

  take $\lim_{n \to \infty} a_n$ but keep in terms of n

  $$\lim_{n \to \infty} \frac{2n}{n^2 - n + 1} =$$

  this approaches ____, so we could use $B = \sum$

-the ratio $\frac{a_n}{b_n} = \frac{\frac{2n}{n^2 - n + 1}}{}$ =

-take the limit of this ratio

$$\lim_{n \to \infty} \frac{}{} = \qquad\qquad = L$$

-from the Limit Comparison Test:

L = ___ is finite and positive, and

$B = \sum$ ____verges,

series $A = \sum_{2}^{\infty} \frac{2n}{n^2 - n + 1}$ therefore ____verges by the

________________________ Test.

5. Does $A = \sum_{1}^{\infty} \frac{1}{2n+1}$ converge or diverge?

Let series $B = \sum$

6. Does $A = \sum_{1}^{\infty} \frac{n2^n}{4n^3}$ converge or diverge?

Let series $B = \sum$