Calculus BC

Section 9.5 - Alternating Series

Obj: - To use the Alternating Series Test to determine the convergence or divergence of a series.
- To classify a convergent series as absolutely or conditionally convergent.

Def: A series in which the terms are alternately positive and negative is called an alternating series.

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-....+\frac{(-1)^{n+1}}{n}+...$$

Alternating Series Theorem (Leibniz's Theorem)

The series $\sum_{1}^{\infty}(-1)^{n+1}a_n \;= a_1 - a_2 + a_3 - a_4 + ...$

converges if **all** three following conditions are satisfied:

a) a_n are all positive

b) $a_n \geq a_{n+1}$ for all n

c) $a_n \to 0$ (abs values of terms are decreasing)

or given $\sum b_n$: alternating, $|b_n| \geq |b_{n+1}|$, and $b_n \to 0$

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Def: If series $\sum a_n$ converges, then

$\sum a_n$ is said to **converge absolutely** if $\sum |a_n|$ also converges.

$\sum a_n$ is said to **converge conditionally** if $\sum |a_n|$ does not converge.

1. State whether $\sum a_n$ converges, if it does, also state whether it converges absolutely or conditionally.

$$\sum a_n = \sum_{1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$$

-Does $\sum a_n$ converge?

- Does $\sum |a_n|$ converge?

i.e. does $\displaystyle\sum_{1}^{\infty} \left| (-1)^{n+1} \frac{1}{n^2} \right| = \sum_{1}^{\infty} \underline{\qquad}$ converge?

- therefore $\sum a_n$ does/does not converge absolutely.

☺ **Absolute Convergence Theorem**

If $\sum |a_n|$ converges, then $\sum a_n$ converges

- By the comparison test, each term of $\sum a_n$ is $\le$ to the corresponding term of $\sum |a_n|$, so therefore, convergence of $\sum |a_n| \Rightarrow$ convergence of $\sum a_n$

2. Does the series $\sum a_n = \sum \frac{\sin n}{n^2}$ converge?

Consider the series

$$\sum |a_n| = \sum \left| \frac{\sin n}{n^2} \right| = \frac{|\sin 1|}{1} + \frac{|\sin 2|}{2^2} + \frac{|\sin 3|}{3^2} + \ldots \quad ,$$

since $|\sin n| \le$ ___ for all n,

we can compare $\sum a_n = \sum \frac{\sin n}{n^2}$ to $B = \sum$

which we know converges/diverges.

Since $\sum |a_n|$ ___ $\sum b_n$

By the ______________ Test, $\sum |a_n|$ ___verges

$\sum a_n = \sum \frac{\sin n}{n^2}$ ___verges by ____________________