Calculus BC

Section 9.6 - The Ratio and Root Tests

Obj: - To use the Ratio Test to determine the convergence or divergence of a series.
- To use the Root Test to determine the convergence or divergence of a series.

When we cannot find a series to do a comparison test, we need to use other methods.

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The Ratio Test

Given the series $\sum a_n$,

Let $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho$

(like the *r* in a geometric series)

then:

a) series converges if $\rho < 1$

b) series diverges if $\rho > 1$

c) cannot tell if $\rho = 1$

e.g. 1+1+1+1+…… diverges ($a_n \to 1$)

but $\sum \frac{1}{n^2}$ converges (integral test)

(both had $\rho = 1$) (try it !)

- The ratio test is effective for series with factorials of n or expressions raised to the nth power:

$$\sum \frac{n!n!}{(2n)!} \quad , \quad \sum \frac{2^n + 5}{3^n} \quad , \quad \sum \frac{4^n \cdot n!n!}{(2n)!}$$

The n-th Root Test

Given the series $\sum a_n$, let $\sqrt[n]{a_n} \to \rho$

a) series converges if $\rho < 1$
b) series diverges if $\rho > 1$
c) cannot tell if $\rho = 1$

1. State whether the series converges or diverges.

$$\sum_{1}^{\infty} \frac{n^2}{2^n}$$

2. State whether the series converges or diverges.

$$\sum \frac{2^n + 5}{3^n}$$

3. State whether the series converges or diverges.

$$\sum \frac{4^n \cdot n!n!}{(2n)!}$$

4. What happens when the Ratio Test is used on

$$\sum_{1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+1} \text{ ?}$$