Calculus BC

<u>Section 9.8 - Power Series</u>

<u>Obj:</u> - To find the radius and interval of convergence of a power series.
- To differentiate and integrate a power series.

<u>Def:</u> A **<u>power series</u>** is a series of the form

$$\sum_{0}^{\infty} a_n x^n \quad \text{or} \quad \sum_{0}^{\infty} a_n (x-c)^n$$

where the center "c " and coefficients a_n are constants.

- **<u>Testing a power series for convergence:</u>**

1. Find all values of x where the series $\sum_{1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ converges (either conditionally or absolutely).

<u>step 1:</u> use the <u>ratio test</u> to find the interval of x where the series <u>converges absolutely,</u>

that is, $\rho < 1$ in the ratio test for $\sum_{1}^{\infty} |a_n| = \sum_{1}^{\infty} \left| (-1)^{n-1} \frac{x^n}{n} \right|$

$$\rho = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$$

$$= \lim_{n\to\infty}\left|\frac{\qquad}{(-1)^{n-1}\,\frac{x^n}{n}}\right|$$

$$= \lim_{n\to\infty}$$

$$= \lim_{n\to\infty}$$

$$\rho = \qquad\qquad\qquad < 1$$

series converge absolutely for ______________

<u>step 2:</u> the series would diverge when $\rho > 1$ or when

> 1

step 3: Now test the series for the endpoints $x=1$ & $x=-1$.

i) for $x=1$, we have the series

$$\sum_{1}^{\infty}(-1)^{n-1}\frac{1^n}{n} \quad \text{or simply} \quad \sum_{1}^{\infty} \_\_\_$$

= - list a few terms

- Do you recognize this series?
- Does it converge?

ii) for $x=-1$, we have the series

$$\sum_{1}^{\infty}(-1)^{n-1}\frac{(-1)^n}{n} \quad \text{or simply} \quad \sum_{1}^{\infty} \_\_\_$$

= - list a few terms

- Do you recognize this series?
- Does it converge?

This series - converges absolutely for $\quad < x <$
- converges conditionally for

The interval of convergence is

The radius of convergence is

2. Find the sum of the series $f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}$

-Look at the derivative of the series:

$$f'(x)=\sum_{n=0}^{\infty}$$

This is a ____________ series with ________

We can find the sum if we keep ________ <1

The series $f'(x)=$

-To get back to $f(x)$, we_________

$$f(x)=$$

$$f(x)= \qquad \text{(equation 1)}$$

To find c, we need to first define the initial conditions.

if $x=0$ then $f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}=$

Equation (1) now becomes

So the series $f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}=$

for $\left|-x^2\right|<1$ or $|x|<1$